MIGHTY LIV:
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Organizers: R. Akhtar, L. DeBiasio, T. Jiang, Z. Miller, D. Pritikin

Tom Bohman

Carnegie Mellon University

Self-correcting estimates for random triangle-removal

Consider the random triangle-removal process. We start with $G(0)$ which is the complete graph on n vertices. Given $G(i)$ let $Q(i)$ be the set of triples xyz such that xy, xz and yz are all edges of $G(i)$. A triple xyz is chosen uniformly at random from $Q(i)$, and we set $G(i+1) = G(i) - \{xy, xz, yz\}$. This process terminates at a triangle-free graph $G(M)$.

In this talk, we discuss dynamic concentration results that show that $G(i)$ closely resembles $G_{n,p}$ with the same edge density. In particular, we show that the number of edges in the final graph $G(M)$ is $n^{3/2+o(1)}$ with high probability. The argument makes us of the fact that key statistics of the process exhibit self-correcting behavior; that is, when one of these variables deviates substantially from its expected trajectory it is subject to a drift that brings it back toward the trajectory.

Joint work with Alan Frieze and Eyal Lubetzky

Vlado Nikiforov

University of Memphis

Spectral extremal problems

For 2-graphs spectral methods have been applied successfully for years, but until recently, these methods have played only a minor role for hypergraphs. This talk will introduce some basic concepts of spectral hypergraph theory. Several extremal problems will be discussed, in particular relations to Turan problems for hypergraphs.

Andrzej Ruciński

Adam Mickiewicz University & Emory University

A double exponential bound on Folkman numbers

For two graphs, G and F, we write $G \longrightarrow F$ if every 2-coloring of the edges of G results in a monochromatic copy of F. A graph G is k-Folkman if $G \longrightarrow K_k$ and $G \not\supset K_{k+1}$. We show that there is a constant $c > 0$ such that for every $k \geq 2$ there exists a k-Folkman graph on at most $2^{k^{ck^2}}$ vertices. Our probabilistic proof is based on a careful analysis of the growth of constants in a modified proof of the result by Rodl and the speaker from 1995 establishing a threshold for the Ramsey property of a binomial random graph $G(n,p)$. Thus, at the same time, we provide a new proof of that result (for two colors) which avoids the use of regularity lemma. This is joint work with Vojta Rodl and Mathias Schacht.

Huseyin Acan

The Ohio State University

Evolution of a Random Permutation Graph

Associated with a permutation σ of $[n]$, there is a graph G_σ whose vertex set is $[n]$ and whose edges correspond to the inversions of σ. Let $\sigma(n, m)$ denote a permutation chosen uniformly at random from all permutations of $[n]$ with m inversions. We find a growth process of a random permutation in which we obtain $\sigma(n, m)$ after the m-th step. We show that the connectedness probability of $G_{\sigma(n,m)}$ is non-decreasing in m. We also find the threshold value of the number of edges to be $\frac{6}{\pi^2} n \ln(n)$ for the connectedness of the inversion graph $G_{\sigma(n,m)}$. This is a joint work with Boris Pittel.

Ariana Angjeli

Oakland University

Linearly many faults in dual-cube-like networks

The dual cube were introduced as better interconnections network than the hypercubes for large scale distributed memory multiprocessors. In this talk we introduce a generalization of these networks, called dual-cube-like networks, which preserve the basic structure of dual cubes and retain many of its topological properties. We investigate structural properties of these networks beyond simple measures such as connectivity. We prove that if up to $kn - (k(k+1))/2$ vertices are deleted from dual-cube-like n-regular network then the resulting graph will either be connected or will have large components and small components having at most $k - 1$ vertices in total,and this result is sharp for $k \leq n$. As an application we derive additional results such as the cyclic vertex-connectivity and the restricted vertex-connectivity of these networks.

Joint work with EDDIE CHENG AND LASZLO LIPTAK.

Deepak Bal

Carnegie Mellon University

Packing Tree Factors in Random and Pseudo-Random Graphs

For a fixed graph H with t vertices, an H-factor of a graph G with n vertices is a collection of vertex disjoint (not necessarily induced) copies of H in G covering all vertices of G. We prove that certain pseudo-random and random graphs may have almost all of their edges covered by a collection of edge disjoint H-factors in the case where H is a tree.

Joint work with Alan Frieze, Michael Krivelevich and Po-Shen Loh.

Peter Blanchard

Grinnell College, University of Iowa

Coloring in Classical Lattices

We prove a Van der Waerden type theorem concerning finite colorings of the Boolean Lattice B_n and the Young Lattice. This is joint project with Amitava Bhattacharya, Tata Institute of Fundamental Research, Mumbai, India.

Axel Brandt

University of Colorado Denver

21st Century Cops and Robbers

Since it's introduction in 1976, pursuit-evasion games on graphs appear in many forms. The well known cops and robbers game in which a 'Cop' attempts to locate a 'Robber' on a graph relates to significant theoretical topics such as treewidth.

We consider a variation introduced by Seager in 2012. Each turn, the Cop 'probes' a vertex v in the pursuit graph G and receives the Robber's current distance from v. The Robber is then permitted to move to any vertex adjacent to his current location, with the exception of v. The Cop wins if at any time he is able to determine the Robber's location exactly.

Seager gave a non-constructive proof that on any tree T, the Cop can win in at most $|T|-2$ probes. In this talk, we give an explicit strategy for the Cop to win on a tree T in at most $|T|-2$ probes, meeting Seager's bound.

This is joint work with Jennifer Diemunsch, Catherine Erbes, Jordan Legrand, and Casey Moffatt.

Arthur Busch

University of Dayton

Toughness and Hamiltonicity in k-trees

A chordal graph is a k-tree if every maximal clique has order $k+1$ and every minimal separating set has order k. In this talk we improve a theorem of Broesma, Xiong & Yoshimoto on toughness in k-trees and consider some related corollaries.

Steve Butler

Iowa State University

Throttling zero forcing propagation speed

Zero forcing is a game played on a graph that starts with a coloring of the vertices as white and black and at each step any vertex colored black with a unique neighbor colored white "forces" the color of the white vertex to be come black. In this talk we look at what happens when we balance the size of the initial set of vertices colored black and the length of time that it takes for all vertices to be colored black. We also give an example that shows it is possible in some graphs to slow down the speed of propagation in the graph by choosing larger initial sets. Joint work with Michael Young.

Ye Chen

West Virginia University

r-hued Coloring of K_4-minor Free Graphs

A list assignment L of G is a mapping that assigns every vertex $v \in V(G)$ a set $L(v)$ of positive integers. For a given list assignment L of G, an (L, r)-coloring of G is a proper coloring c such that for any vertex v with degree $d(v)$, $c(v) \in L(v)$ and v is adjacent to at least $\min\{d(v), r\}$ different colors. The *r-hued chromatic number* of G, $\chi_r(G)$, is the least integer k such that for any $v \in V(G)$ with $L(v) = \{1, 2, \cdots, k\}$, G has an (L, r)-coloring. The *r-hued list chromatic number* of G, $\chi_{L,r}(G)$, is the least integer k such that for any $v \in V(G)$ and every list assignment L with $|L(v)| = k$, G has an (L, r)-coloring. Let $K(r) = r + 3$ if $2 \leq r \leq 3$, and $K(r) = \lfloor 3r/2 \rfloor + 1$ if $r \geq 4$. We proved that if G is a K_4-minor free graph, then

(i) $\chi_r(G) \leq K(r)$, and the bound can be attained;

(ii) $\chi_{L,r}(G) \leq K(r) + 1$.

Zhihong Chen

Butler University

Eulerian-connected Graphs

A graph G is Eulerian-connected if for any u and v in $V(G)$, G has a spanning (u, v)-trail. A graph G is edge-Eulerian-connected if for any e' and e'' in $E(G)$, G has a spanning (e', e'')-trail. For an integer $r \geq 0$, a graph is called r-Eulerian-connected if for any $X \subseteq E(G)$ with $|X| \leq r$, and for any $u,\ v \in V(G)$, G has a spanning (u, v)-trail T such that $X \subseteq E(T)$. The r-edge-Eulerian-connectivity of a graph can be defined similarly. Let $\theta(r)$ be the minimum value of k such that every k-edge-connected graph is r-Eulerian-connected. Catlin proved that $\theta(0) = 4$. In this talk, we shall show that $\theta(r) = 4$ for $0 \leq r \leq 2$, and $\theta(r) = r + 1$ for $r \geq 3$, and show some new results on r-edge-Eulerian connectivity.

Ilkyoo Choi

University of Illinois at Urbana-Champaign

On Choosability with Separation of Planar Graphs with Forbidden Cycles

We study choosability with separation which is a constrained version of list coloring of graphs. A (k, d)-*list assignment* L of a graph G is a function that assigns to each vertex v a list $L(v)$ of at least k colors and for any adjacent pair xy, the lists $L(x)$ and $L(y)$ share at most d colors. A graph G is (k, d)-choosable if there exists an L-coloring of G for every (k, d)-list assignment L. This concept is also known as choosability with separation. We prove that planar graphs without 4-cycles are $(3, 1)$-choosable and that planar graphs without 5-cycles and 6-cycles are $(3, 1)$-choosable. In addition, we give an alternative and slightly stronger proof that triangle-free planar graphs are $(3, 1)$-choosable.

John Engbers

University of Notre Dame

Extremal H-colorings of forests and trees

Given a finite graph H, an H-coloring of a finite, simple graph G (or graph homomorphism) is a map from the vertices of G to the vertices of H that preserves edge adjacency. H-colorings generalize many important graph theoretic notions, such as proper q-colorings (via $H = K_q$) and independent sets (via H as an edge with a loop on one endvertex).

Given a family of graphs $\mathcal{G}$ and a graph H, which graph(s) in $\mathcal{G}$ have the largest number of H-colorings? We present several results for the family of n-vertex forests and the family of n-vertex trees. Numerous open questions remain.

Lisa Espig

Carnegie Mellon University

Threshold for Zebraic Hamilton Cycles in Random Graphs

When studying random graphs, we often want to know the threshold for the emergence of a particular structure. In other words, in the Erdös-Rényi random graph model $G_{n,p}$ - where we consider a graph on n vertices with each edge appearing randomly with probability p, how large must p be in order to guarantee a matching in the graph with high probability? This and other thresholds are well-studied. Here we find the threshold for a different kind of structure. Namely, we will find the threshold for which a randomly 2-colored random graph contains a zebraic Hamilton cycle - one whose edges alternate between the two colors.

David Galvin

University of Notre Dame

Twin conventions and graph Stirling numbers

Recently Griffiths asked (and answered) the question: in how many ways can n sets of twins at a twin convention break into non-empty groups, with no group allowed to contain a pair of twins? One approach to this problem leads naturally to the notion of a "graph Stirling number of the second kind". I'll say what we know about these numbers and what we don't, and then use them to address Griffith's question with "twins" replaced by "triplets", "quadruplets", etc. Partly joint work with Do Trong Thanh.

Sogol Jahanbekam

University of Illinois at Urbana-Champaign

Rainbow Spanning Subgraphs in Edge-Colored Complete Graphs

For integers n and t, let $r(n,t)$ be the maximum number of colors in an edge-coloring of the complete graph K_n that does not contain t edge-disjoint rainbow spanning trees. Let $s(n,t)$ be the maximum number of colors in an edge-coloring of K_n containing no rainbow spanning subgraph with diameter at most t. We determine $r(n,t)$ for $n > 2t + \sqrt{6t - \frac{23}{4}} + \frac{5}{2}$ and for $n = 2t$. We also determine $s(n,t)$ for all integers n and t. This is joint work with Douglas B. West.

Jaehoon Kim

University of Illinois at Urbana-Champaign

(0,1)-improper coloring of sparse triangle-free graph

A graph G is a $(0,1)$-colorable if $V(G)$ can be partitioned into two sets V_0 and V_1 so that $G[V_0]$ is an independent set and $G[V_1]$ has maximum degree at most 1. The problem of verifying whether a graph is $(0,1)$-colorable is NP-complete even in the class of planar graphs of girth 9.

Maximum average degree, $Mad(G) = \max_{H \subset G}\{\frac{2|E(H)|}{|V(H)|}\}$, is a graph parameter measuring how sparse the graph G is. Borodin and Kostochka showed that every graph G with $Mad(G) \leq \frac{12}{5}$ is $(0,1)$-colorable, thus every planar graph with girth at least 12 also is $(0,1)$-colorable.

The aim of this talk is to prove that every triangle-free graph G with $Mad(G) \leq \frac{22}{9}$ is $(0,1)$-colorable. We prove the slightly stronger statement that every triangle-free graph G with $|E(H)| < \frac{11|V(H)|+5}{9}$ for every subgraph H is $(0,1)$-colorable and show that there are infinitely many not $(0,1)$-colorable graphs G with $|E(G)| = \frac{11|V(G)|+5}{9}$. This is joint work with A. V. Kostochka and Xuding Zhu.

Bernard Lidicky

University of Illinois at Urbana-Champaign

Coloring planar graphs with 4-triangles

A sharpening of Grötzsch Theorem, the Grünbaum-Axenov Theorem, states that every planar graph with at most three triangles is 3-colorable. It is best possible since not all planar graphs with four triangles are 3-colorable. In this talk, we discuss 3-colorability of planar graphs with four triangles.

This is joint work with O. V. Borodin, Z Dvořák, A. Kostochka, and M. Yancey

Tom Mahoney

University of Illinois at Urbana-Champaign

Online sum list coloring of graphs

In *online list coloring* (introduced by Zhu and by Schauz in 2009), on each round the set of vertices having a particular color in their lists is revealed, and the coloring algorithm chooses an independent subset to receive that color. A graph G is said to be f-paintable for a function $f : V(G) \to \mathbb{N}$ if there is an algorithm to produce a successful coloring when each vertex v is allowed to be presented at most $f(v)$ times.

In 2002 Isaak introduced *sum list coloring* and the resulting parameter called *sum-choosability*. The *online sum-choosability*, or *sum-paintability*, of G is the least $\sum f(v)$ over all functions f such that G is f-paintable; this value is denoted by $\chi_{sp}(G)$.

The *generalied theta-graph* $\Theta_{\ell_1,\ldots,\ell_k}$ consists of two vertices joined by internally disjoint paths of lengths $\ell_1, \ldots, \ell_k$. Strengthening results of Berliner et al., we show that the sum-paintability of G depends only on the sum-paintability of its blocks, and we prove $\chi_{sp}(K_{2,r}) = 2r + \min\{l+m : lm > r\}$. The *book* B_r is the graph $\Theta_{1,2,\ldots,2}$ with r internally disjoint paths of length 2. We prove $\chi_{sp}(B_r) = 2r + \min\{l+m : m(l-m) + \binom{m}{2} > r\}$. Using these results, we determine the sum-paintability for all generalized theta-graphs.

Zeinab Maleki

University of Illinois at Urbana-Champaign & Isfahan University of Technology

Some lower bounds for the intersection dimension of graphs

The *intersection dimension* of a graph with respect to a set of non-negative integers L is the small number l for which there is an assignment on the vertices to subsets $A_v \subseteq \{1, \ldots, l\}$, such that every two vertices u, v are adjacent if and only if $|A_u \cap A_v| \in L$. The *bipartite intersection dimension* is defined similarly when the conditions are considered only for the vertices in different parts. The *absolut dimension* of a graph G is the minimum intersection dimension of G over all sets L. Finding graphs with large (bipartite) absolute dimension would have important consequences in the complexity theory. In this talk, we present some lower bounds for the (bipartite) intersection dimension of a graph with respect to various types L in terms of the minimum rank of graph. (This is a joint work with Behnaz Omoomi)

Daniel McDonald

University of Illinois at Urbana-Champaign

On-line rank number of trees

A k-ranking of a graph G is a labeling of its vertices from $[k]$ such that any nontrivial path whose endpoints have the same label contains a larger label. The least k for which G has a k-ranking is the rank number of G, also known as tree depth. Applications of rankings include VLSI design, parallel computing, and factory scheduling. The on-line ranking problem asks for an algorithm for ranking the vertices of G as they are presented one at a time along with all previously ranked vertices and the edges between (so each vertex is presented as the lone unranked vertex in a partially labeled induced subgraph of G whose final placement in G is not specified). The on-line rank number of G is the minimum over all such algorithms of the largest label that algorithm can be forced to use. We give upper and lower bounds on the on-line rank number of trees in terms of maximum degree, diameter, and other structural parameters.

Terry McKee

Wright State University

Weighty Characterizations of Two Graph Classes

Defining the weight of a complete subgraph to be the number of maxcliques that contain it, I give a stylish characterization of strongly chordal graphs in terms of their clique graphs, and a related-but-quirky characterization of trivially perfect graphs.

Darren Parker

Grand Valley State University

Multidesigns for Graph-Triples of Order 6

We call $T = (G_1, G_2, G_3)$ a *graph-triple of order* t if the G_i are pairwise non-isomorphic graphs on t non-isolated vertices whose edges can be combined to form K_t. If $m \geq t$, we say T *divides* K_m if $E(K_m)$ can be partitioned into copies of the graphs in T with each G_i used at least once, and we call such a partition a T-*multidecomposition*. In this talk, we determine T-multidecompositions of complete graphs, where T is a graph-triple of order 6. Moreover, we determine maximum multipackings and minimum multicoverings when K_m does not admit a multidecomposition.

Nicholas Peterson

The Ohio State University

On Random k-Out Graphs with Preferential Attachment

We generalize a model of Hansen and Jaworski for random mappings which exhibit preferential attachment to a mapping $[n] \mapsto [n]^k$ - or, equivalently, a random digraph $M^{\alpha}_{n,k}$ with labeled arcs and uniform out-degree k. Each vertex starts with some weight $\alpha > 0$; each vertex chooses k images, one at a time in a fixed order, with probability of choosing a given vertex proportional to its current weight; and the weight of the chosen vertex increases by 1 before the next decision is made. The limiting case $M^{\infty}_{n,k}$ is a uniformly random k-out digraph with labeled arcs.

We establish a limiting distribution for the vertex connectivity of the graph obtained from $M^{\alpha}_{n,k}$ by ignoring loops, multiple edges, and arc directions. We also seek to answer the question: how fast must α grow (relative to n) in order to make the difference between $M^{\alpha}_{n,k}$ and $M^{\infty}_{n,k}$ asymptotically negligible? Measuring with the total variation distance, we establish $\alpha = \Theta(\sqrt{n})$ as a sharp threshold for this behavior.

This is a joint work with Boris Pittel.

Daniel Poole

The Ohio State University

Weak Hamilton Cycles in Random Hypergraphs

We say that a hypergraph, H, has a weak Hamilton cycle if there is some cyclic ordering of the vertices of H, such that each consecutive pair of vertices is contained in some hyperedge. We find the sharp threshold for the existence of a weak Hamilton cycle in the random d-uniform hypergraph, $H_d(n,m)$, on n vertices with m hyperedges. While the Erdős-Rényi graph $G(n,m) = H_2(n,m)$ with high probability develops a Hamilton cycle when the minimum vertex degree reaches 2, for $d > 2$ $H_d(n,m)$ whp becomes Hamiltonian sooner, when the minimum vertex degree reaches 1. Our proofs use hypergraph analogues of Posá's lemma and De la Vega's Theorem.

Gregory Puleo

University of Illinois at Urbana-Champaign

Tuza's Conjecture for Graphs of Max Average Degree Less than 7

Suppose I wish to make a graph G triangle-free by removing a small number of edges. An obvious obstruction is the presence of a large set of edge-disjoint triangles, since I must remove one edge from each triangle. On the other hand, removing all the edges in a maximal set of edge-disjoint triangles clearly makes G triangle-free. *Tuza's conjecture* states that the worst-case number of edges that must be removed is somewhere between these extremes: if $\tau(G)$ is the number of edges that must be removed to make G triangle-free and $\nu(G)$ is the maximum number of edge-disjoint triangles in G, then Tuza's conjecture states that $\tau(G) \leq 2\nu(G)$. Using the method of discharging, we show that Tuza's conjecture holds whenever the maximum average degree $\mathrm{Mad}(G) < 7$. This subsumes several earlier results and represents the first application of discharging to this problem.

Michael Santana

University of Illinois at Urbana-Champaign

Pairs of forbidden subgraphs for pancyclicity

The Matthews-Sumner Conjecture claims that every 4-connected, claw-free graph contains a Hamiltonian cycle. This conjecture has been the impetus for a great deal of research into structural properties of claw-free graphs, and in particular, pancyclicity. While it is known that forbidding the claw is an insufficient condition to imply pancyclicity of 4-connected graphs, we look to forbid more substructures to obtain this end. In this talk we will present a characterization of the forbidden subgraphs that imply pancyclicity for 4-connected, claw-free graphs. This answers a question of Gould and extends a result of Gould, Łuczak, and Pfender for 3-connected graphs. We will also present some interesting questions continuing this theme.

This is joint work with James Carraher, Michael Ferrara, and Timothy Morris.

Robert Seiver

Miami University

Set Families without Chain-traces

Given a family $\mathcal{F}$ of subsets of a set X and $W \subseteq X$, the *trace* of $\mathcal{F}$ on W, denoted by $\mathcal{F}|_W$ is defined to be $\mathcal{F}|_W = \{F \cap W : F \in \mathcal{F}\}$. If for some k-set W in X, $\mathcal{F}|_W$ contains a maximal chain $C_0 \subset C_1 \cdots \subset C_k$ with $|C_i| = i$, then we say that $\mathcal{F}$ has a *k-chain-trace*. For fixed k and sufficiently large n, Patkós showed that every family $\mathcal{F}$ of k-subsets of $[n]$ with more than $\binom{n-1}{k-1}$ members has a k-chain-trace. The bound is best possible. Patkós made a more general conjecture that for fixed $m \geq k \geq 2$ and sufficiently large n, every family $\mathcal{F}$ of m-subsets of $[n]$ with more than $\binom{n-m+k-1}{k-1}$ members has a k-chain-trace. We prove Patkós' conjecture and for large n also establish strong stability property of the unique extremal family.

This is joint work with Tao Jiang.

Keke Wang

West Viriginia University

Cycle chains and Hamiltonian 3-connected claw-free graphs

We develop a cycle chain method to prove that every 3-edge-connected graph G is supereulerian if every

3-edge cut of G intersects with short cycles in G. This is applied to the study of Hamiltonian claw-free graphs, and provide a unified treatment with short proofs for several former results. New sufficient conditions for Hamiltonian claw-free graphs are also obtained.

Stephen Young

University of Louisville

An Alon-Boppana result for the normalized Laplacian

We prove a Alon-Boppana style bound for the spectral gap of the normalized Laplacian for general graphs via a bound on the weighted spectral radius of the universal cover graph.

Minquan Zhan

Millersville University

The Discharging Method and 3-Connected Essentially 10-Connected Line Graphs

We use the discharging method to prove that every 3-connected, essentially 10-connected line graph is hamiltonian connected.

Meng Zhang

West Virginia University

Supereulerian Graphs and The Petersen Graph

A graph G is supereulerian if G has a spanning eulerian subgraph. Boesch et al proposed the problem of characterizing supereulerian graphs. In this talk I will present any 3-edge-connected graph with at most 11 edgecuts of size 3 is supereulerian if and only if it cannot be contractible to the Petersen graph.

Xiangqian "Joe" Zhou

Wright State University

Unavoidable minors of large 4-connected bicircular matroids

The bicircular matroid of a graph G is the matroid with ground set E(G) and a subset is independent iff each connected component of the subgraph spanned by that set contains at most one cycle. In this talk, we present a recent result on unavoidable minors of large 4-connected bicircular matroids. This is joint work with Daniel Slilaty, Deborah Chun, and Tyler Moss.

Uta Ziegler

Western Kentucky University

Embedding of 4-regular planar graphs on a grid

The author and her collaborators have developed methods to embed 4-regular planar graphs into a 2-dimensional grid while preserving the topology (i.e the cyclic order of vertices at each vertex). We call a 4-regular planar graph algebraic if it can be reduced to a trivial graph (consisting of one 4-regular vertex) by the repeated collapses of digons. Here we present a special case of our algorithm for algebraic 4-regular graphs, where it can be shown that the length of a constructed embedding is linear in the number of vertices of the original graph. These graph embedding results can be applied to obtain results about the rope length of knots.

10:50-11:10, Room 153

Ping Hu

University of Illinois at Urbana-Champaign

Phase transitions in the Ramsey-Turán theory

Let $f(n)$ be a function and L be a graph. Denote by $RT(n, L, f(n))$ the maximum number of edges of an L-free graph on n vertices with independence number less than $f(n)$. Erdős and Sós asked if $RT\left(n, K_5, c\sqrt{n}\right) = o(n^2)$ for some constant c. We answer this question by proving the stronger $RT\left(n, K_5, o\left(\sqrt{n \log n}\right)\right) = o(n^2)$. It is known that $RT\left(n, K_5, c\sqrt{n \log n}\right) = n^2/4 + o(n^2)$ for $c > 1$, so one can say that K_5 has a Ramsey-Turán phase transition at $c\sqrt{n \log n}$. We extend this result to several other K_s's and functions $f(n)$, determining many more phase transitions. We shall formulate several open problems, in particular, whether variants of the Bollobás-Erdős graph exist to give good lower bounds on $RT\left(n, K_s, f(n)\right)$ for various pairs of s and $f(n)$. Among others, we use Szemerédi's Regularity Lemma and the Hypergraph Dependent Random Choice Lemma. We also present a short proof of the fact that K_s-free graphs with small independence number are sparse. Joint work with Jozsef Balogh and Miklos Simonovits.